Change + Conservation of $\vec{L}, \vec{H}, E$ for a particle

Start with $\vec{F} = m\vec{a} = \vec{F}(t, \vec{r}, \vec{v})$

We want to know properties of solutions

Why? 1. To check solutions numerically or analytically

2. Can use $\vec{L}, \vec{H}$, + E to solve simple problems

Linear Momentum (L)

$\vec{F} = m\vec{a}$ $\qquad \vec{a} = \frac{d\vec{v}}{dt}$ $\qquad \dot{\vec{L}} = \frac{d}{dt}(m\vec{v})$

$\vec{F} = \dot{\vec{L}}$

$\int_{t_1}^{t_2} \vec{F}\,dt = \Delta\vec{L}$ $\qquad \boxed{\text{Principle of Impulse-Momentum}}$

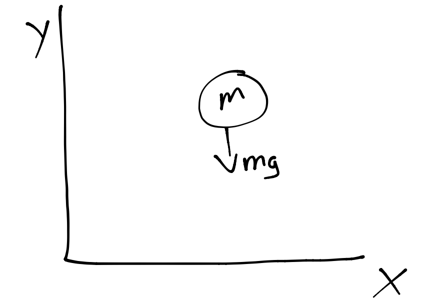
Impulse $= \int_{t_1}^{t_2} \vec{F}\,dt$ $\qquad$ Where Linear momentum $= m\vec{v}$

Special Case: $\vec{F} = 0$ $\qquad \int \vec{F}\,dt = 0,\ \Delta L = 0$

$\vec{L} = \text{constant}$

"Conservation of linear momentum"

Ex:



LMB: $\vec{F} = m\vec{a} = -mg\,\hat{j} = \dot{\vec{L}}$

can we use conservation of linear momentum in this case?

- yes, break it into components

$$\boxed{\begin{array}{l} \vec{F} = ma \cdot \hat{i} \\ L_x = \text{constant} \end{array}}$$

conservation of linear momentum in the x-direction

<u>Angular Momentum</u> Everything from linear momentum balance is a consequence of angular momentum balance

$\vec{F} = m\vec{a}$ $\qquad \vec{r} \times \vec{F} = \vec{r} \times (m\vec{a}) = m\,\vec{r} \times \vec{a}$

$\vec{r} \to r_{p/o}$

Let's look at $\frac{d}{dt}(\vec{r} \times \vec{v})$ — really a guess

$= \dot{\vec{r}} \times \vec{v} + \vec{r} \times \dot{\vec{v}}$

$= 0 + \vec{r} \times \vec{a} = \vec{r} \times \vec{a}$ ✓

$\vec{r} \times \vec{F} = \frac{d}{dt}(\vec{r} \times m\vec{v}) = \vec{H} \longrightarrow \vec{L} =$ book notation

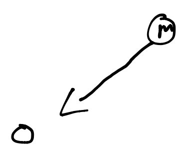
$\vec{r} \times \vec{F}\,dt = \int \vec{M}\,dt = \Delta\vec{H}$

$\vec{M}$: moment vector

Special Case $\vec{M} = 0$ (forces have no moments)

$\vec{H} =$ constant (conservation of angular momentum)

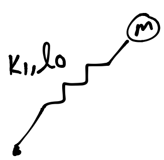
Ex: central force: $\vec{F} = F \dfrac{\vec{r}}{|\vec{r}|} = F\hat{e}_r$



$$\vec{F} = \frac{-mMG}{r^2}\hat{e}_r = \frac{-mMG}{r^3}\vec{r}$$

Ex: spring

$$\vec{F}_{spring} = -K(\Delta l)\,\hat{e}_r\,, \quad \Delta l = |\vec{r}| - l_0$$



$$= -K(|\vec{r}| - l_0)\hat{e}_r = -K\left(\frac{|\vec{r}| - l_0}{|\vec{r}|}\right)\vec{r}$$

$\vec{r} \times m\vec{v}$
$=$ constant

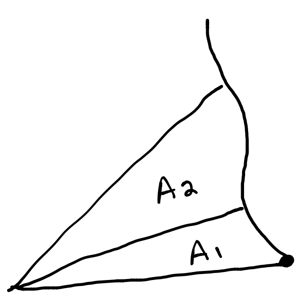
<u>Comments on Conservation of Angular Momentum</u>

$\vec{M} = 0 \qquad \vec{H} =$ constant

$\vec{M}_O \qquad \vec{H}_O$

$\vec{r} \times m\vec{v} =$ constant

"Equal Areas in Equal Times"



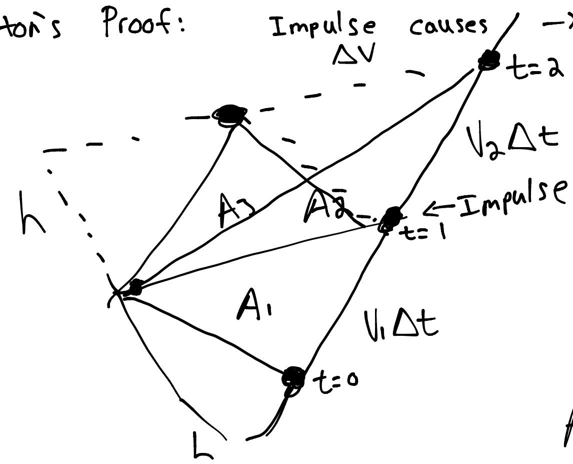
$$\Delta A = \tfrac{1}{2} r \cdot v^{\perp}\, dt$$
$$= \tfrac{1}{2}|\vec{r} \times \vec{v}|$$
$$\dot{A} = \text{constant}$$

if $\Delta t_1 = \Delta t_2$

$A_1 = A_2$

Feynman Messenger Lectures
"Character of Physical Law"

Newton's Proof:



→ path of particle in straight line at constant speed

$A_1 = A_2 = h V_1 \Delta t$

↳ V_2

$A_2 = A_3 = r_2 h_2$

$A_3 = A_1$

central force has no effect on Area swept

<u>Energy</u> - review calc/math before next class

5 equivalent facts about Vector field $\vec{F}$

If 1 is true, all are true. If 1 is false, all are false

1. $\vec{F}$ is conservative

2. $\vec{F} = \vec{\nabla} V$ $\quad$ V is a single valued potential

3. $\vec{\nabla}\,\vec{F} = 0$ everywhere

4. $\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r} =$ path independent

5. $\oint \vec{F}\cdot d\vec{r} = 0$ all closed loops